

# Observable effects from extra dimensions

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## Abstract

For any multidimensional theory with compactified internal spaces, conformal excitations of the internal space metric result in gravitational excitons in the external spacetime. These excitations contribute either to dark matter or to cross sections of usual particles.

The large-scale dynamics of the observable part of our present time universe is well described by the Friedmann model with four-dimensional Friedmann-Robertson-Walker metric. However, it is possible that spacetime at short (Planck) distances might have a dimensionality of more than four and possess a rather complex topology. String theory and its recent generalizations —  $p$ -brane,  $M$ - and  $F$ -theory widely use this concept and give it a new foundation. Most of these unified models are initially constructed on a higher-dimensional spacetime manifold, say of dimension  $D > 4$ , which then undergoes some scheme of spontaneous compactification yielding a direct product manifold  $M^4 \times K^{D-4}$  where  $M^4$  is the manifold of spacetime and  $K^{D-4}$  is a compact internal space. Hence it is natural to investigate observable consequences of such a compactification hypothesis.

One of the main problems in multidimensional models consists in the dynamical process leading from a stage with all dimensions developing on the same scale to the actual stage of the universe, where we have only four external dimensions and all internal spaces have to be compactified and contracted to sufficiently small scales, so that they are apparently unobservable. To make the internal dimensions unobservable at the actual stage of the universe we have to demand their contraction to scales  $10^{-17} \text{ cm} - 10^{-33} \text{ cm}$  (between the Fermi and Planck lengths). This leads to an effectively four-dimensional universe. However, there is still a question on possible observable phenomena following from such small compactified spaces. In the present paper we predict some physical effects which should take place in this case.

As starting point let us consider a simple multidimensional cosmological model (MCM) with spacetime manifold

$$M = M_0 \times M_1 \times \dots \times M_n \quad (1)$$

and with decomposed metric on  $M$

$$g = g_{MN}(X) dX^M \otimes dX^N = g^{(0)} + \sum_{i=1}^n e^{2\beta^i(x)} g^{(i)}, \quad (2)$$

where  $x$  are some coordinates of the  $D_0 = d_0 + 1$  - dimensional manifold  $M_0$  and

$$g^{(0)} = g_{\mu\nu}^{(0)}(x) dx^\mu \otimes dx^\nu. \quad (3)$$

Let the manifolds  $M_i$  be  $d_i$ -dimensional Einstein spaces with metric  $g^{(i)} = g_{m_i n_i}^{(i)}(y_i) dy_i^{m_i} \otimes dy_i^{n_i}$ , i.e.,

$$R_{m_i n_i} [g^{(i)}] = \lambda^i g_{m_i n_i}^{(i)}, \quad m_i, n_i = 1, \dots, d_i \quad \text{and} \quad R [g^{(i)}] = \lambda^i d_i \equiv R_i. \quad (4)$$

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In the case of constant curvature spaces parameters  $\lambda^i$  are normalized as  $\lambda^i = k_i(d_i - 1)$  with  $k_i = \pm 1, 0$ . The internal spaces  $M_i$  ( $i = 1, \dots, n$ ) may have nontrivial global topology, being compact (i.e. closed and bounded) for any sign of spatial topology.

With total dimension  $D = 1 + \sum_{i=0}^n d_i$ ,  $\kappa^2$  a  $D$ -dimensional gravitational constant,  $\Lambda$  - a  $D$ -dimensional bare cosmological constant we consider an action of the form

$$S = \frac{1}{2\kappa^2} \int_M d^D X \sqrt{|g|} \{R[g] - 2\Lambda\}. \quad (5)$$

Of course, the ansatz of our model with this action is rather simplified and can describe only partial aspects of a more realistic theory. The  $\Lambda$  - term can originate, for example, from  $D - 1$  - form gauge fields [1]. We also could enlarge the model action e.g. by inclusion of a dilatonic scalar field as well as other matter fields, take into account the Casimir effect due to non-trivial topology of the manifold (1), consider different monopole ansätze etc. However, to reveal observable effects following from extra dimensions it is sufficient to consider such a simplified model.

After dimensional reduction and a conformal transformation to the Einstein frame

$$g_{\mu\nu}^{(0)} = \Omega^2 \tilde{g}_{\mu\nu}^{(0)} = \exp\left(-\frac{2}{D_0 - 2} \sum_{i=1}^n d_i \beta^i\right) \tilde{g}_{\mu\nu}^{(0)} \quad (6)$$

the action reads [2]

$$S = \frac{1}{2\kappa_0^2} \int_{M_0} d^{D_0} x \sqrt{|\tilde{g}^{(0)}|} \left\{ \tilde{R}[\tilde{g}^{(0)}] - \tilde{G}_{ij} \tilde{g}^{(0)\mu\nu} \partial_\mu \beta^i \partial_\nu \beta^j - 2U_{eff} \right\}, \quad (7)$$

where  $\kappa_0^2 = \kappa^2/V_I$  is the  $D_0$ -dimensional gravitational constant and  $V_I = \prod_{i=1}^n v_i = \prod_{i=1}^n \int_{M_i} d^{d_i} y \sqrt{|g^{(i)}|}$  defines the internal space volume corresponding to the scale factors  $a_i \equiv 1, i = 1, \dots, n$ . The tensor components of the midisuperspace metric (target space metric on  $\mathbb{R}_T^n$ )  $\tilde{G}_{ij}$  ( $i, j = 1, \dots, n$ ) and the effective potential are respectively

$$\tilde{G}_{ij} = d_i \delta_{ij} + \frac{1}{D_0 - 2} d_i d_j \quad (8)$$

and

$$U_{eff} = \left( \prod_{i=1}^n e^{d_i \beta^i} \right)^{-\frac{2}{D_0 - 2}} \left[ -\frac{1}{2} \sum_{i=1}^n R_i e^{-2\beta^i} + \Lambda + \kappa^2 \rho \right], \quad (9)$$

where the phenomenological energy density  $\kappa^2 \rho$  is not specified here. Depending on the concret model [2] it takes into account e.g. the Casimir effect of additional matter fields or Freund-Rubin monopoles. In the case of a purely geometrical model it vanishes.

Variation of action (7) with respect to  $\tilde{g}^{(0)}$  and  $\beta^i$  gives the Euler-Lagrange equations for the scale factors and the external metric. Assuming that there exists a well defined splitting of the physical fields  $(\tilde{g}^{(0)}, \beta)$  into not necessarily constant background components  $(\bar{g}, \bar{\beta})$  and small perturbational (fluctuation) components  $(h, \eta)$

$$\begin{aligned} \tilde{g}_{\mu\nu}^{(0)} &= \bar{g}_{\mu\nu} + h_{\mu\nu}, \\ \beta^i &= \bar{\beta}^i + \eta^i \end{aligned} \quad (10)$$

we can perform a perturbational analysis of the interaction dynamics. For example, for the internal space scale factors we obtain in zeroth and first order approximation respectively [3]

$$\square \bar{\beta}^i = [\bar{G}^{-1}]^{ij} b_j(\bar{\beta}), \quad (11)$$

$$\square \eta^i - [\bar{G}^{-1}]^{ij} A_{jk}(\bar{\beta}) \eta^k = \frac{1}{\sqrt{|\bar{g}|}} \partial_\nu \left( \sqrt{|\bar{g}|} h^{\mu\nu} \partial_\mu \bar{\beta}^i \right) - \frac{1}{2} \bar{g}^{\mu\nu} \partial_\mu \bar{\beta}^i \partial_\nu h, \quad (12)$$

where we abbreviate

$$A_{ij} := \frac{\partial^2 U_{eff}}{\partial \beta^i \partial \beta^j}, \quad b_i := \frac{\partial U_{eff}}{\partial \beta^i}. \quad (13)$$

In equations (11) and (12) the covariant derivative is taken with respect to the background metric  $\bar{g}_{\mu\nu}$  and indices in  $h_{\mu\nu}$  are raised and lowered by the background metric  $\bar{g}_{\mu\nu}$ , e.g.  $h = h_{\mu\nu} \bar{g}^{\mu\nu}$ .

We can diagonalize matrix  $[\bar{G}^{-1} A]_k^i \equiv [\bar{G}^{-1}]^{ij} A_{jk}(\bar{\beta})$  by an appropriate background depending  $SO(n)$  - rotation  $S = S(\bar{\beta})$

$$S^{-1} \bar{G}^{-1} A S \stackrel{def}{=} M^2 = \text{diag}[m_1^2(\bar{\beta}), \dots, m_n^2(\bar{\beta})] \quad (14)$$

and rewrite Eq. (12) in terms of generalized normal modes (gravitational excitons [2])  $\psi = S^{-1}\eta$ :

$$\bar{g}^{\mu\nu} D_\mu D_\nu \psi - M^2(\bar{\beta})\psi = \left( h^{\mu\nu} - \frac{1}{2} \bar{g}^{\mu\nu} h \right)_{;\nu} D_\mu \bar{\varphi} + h^{\mu\nu} D_\mu D_\nu \bar{\varphi}, \quad (15)$$

where  $\bar{\varphi}$  are  $SO(n)$ -rotated background scale factors  $\bar{\varphi} = S^{-1}\bar{\beta}$  and  $M^2$  can be interpreted as background depending diagonal mass matrix for the gravitational excitons.

$D_\mu$  denotes a covariant derivative

$$D_\mu := \partial_\mu + \Gamma_\mu + \omega_\mu, \quad \omega_\mu := S^{-1} \partial_\mu S \quad (16)$$

with  $\Gamma_\mu + \omega_\mu$  as connection on the fibre bundle  $E(M_0, \mathbb{R}^{D_0} \oplus \mathbb{R}_T^n) \rightarrow M_0$  consisting of the base manifold  $M_0$  and vector spaces  $\mathbb{R}_x^{D_0} \oplus \mathbb{R}_{Tx}^n = T_x M_0 \oplus \{(\eta^1(x), \dots, \eta^n(x))\}$  as fibres. So, the background components  $\bar{\beta}^i(x)$  via the effective potential  $U_{eff}$  and its Hessian  $A_{ij}(\bar{\beta})$  play the role of a medium for the gravitational excitons  $\psi^i(x)$ . Propagating in  $M_0$  filled with this medium they change their masses as well as the direction of their "polarization" defined by the unit vector in the fibre space

$$\xi(x) := \frac{\psi(x)}{|\psi(x)|} \in S^{n-1} \subset \mathbb{R}^n, \quad (17)$$

where  $S^{n-1}$  denotes the  $(n-1)$ -dimensional sphere.

From (12) and (15) we see that in the lowest order (linear) approximation of the used perturbation theory a non-constant scale factor background is needed for an interaction between gravitational excitons and gravitons. This can be also easily seen from the interaction term in the action functional (in the lowest order approximation and in traceless gauge:  $h = 0$ )

$$S_{int} = \frac{1}{\kappa_0^2} \int_{M_0} d^{D_0} x \sqrt{|\bar{g}|} h^{\mu\nu} \bar{G}_{ij} \partial_\mu \bar{\beta}^i \partial_\nu \eta^j. \quad (18)$$

For constant scale factor backgrounds  $\bar{\beta} = const$  the system is necessarily located in one of the minima  $\bar{\beta} = \beta_{(c)}$  of the effective potential  $U_{eff}$  so that  $b_i(\beta_{(c)}) = 0$ , and  $U_{eff}(\beta_{(c)}) = \Lambda_{(c)eff}$  plays the role of a  $D_0$ -dimensional effective cosmological constant (according to recent observational data there is a strong evidence for a positive cosmological constant of the universe [4]). Gravitational excitons and gravitons can in this case only interact via nonlinear (higher order) terms. In the linear approximation they decouple over constant scale factor backgrounds due to vanishing terms in (12) and (15) (see also (18)). This means that in this case conformal excitations of the metric of the internal spaces behave as non-interacting massive scalar particles developing on the background of the external spacetime. Due to their vanishing cross section they will contribute only to dark matter.

From the geometrical point of view it is clear that gravitational excitons are an inevitable consequence of the existence of extra dimensions. For any theory with compactified internal spaces conformal excitations of the internal space metric will result in gravitational excitons in the external spacetime. The form of the effective potential as well as masses of gravitational excitons and the value of the effective cosmological constant are model dependent. It is important to note that even for internal spaces compactified at Planck scales the masses of gravitational excitons can run a very wide range of values — from very heavy (of order of the Planck mass) to extremely light. It depends on the parameters of a the concrete model. For example, models with one internal factor space and nonvanishing energy density  $\kappa^2 \rho \neq 0$  induced e.g. by Freund-Rubin monopoles or the Casimir effect of additional matter fields, or with vanishing  $\kappa^2 \rho \equiv 0$ , yield exciton masses which are up to a numerical prefactor of order one [2]

$$m \sim \left( a_{(c)}^{-1} \right)^{\frac{D-2}{2}}. \quad (19)$$

Here we assumed  $D_0 = 4$ , and  $a_{(c)}$  is the compactification scale of the internal factor space. This means that if compactification takes place at  $a_{(c)} = 10^2 L_{Pl}$  then  $m \sim 10^{-8} m_{Pl}$  and  $m \sim 10^{-24} m_{Pl}$  for  $D = 10$  and  $D = 26$  correspondingly. Of course, gravitational excitons can be excited at the present time if their masses are much less than the Planck mass. So, even for compactification scales very close to the Planckian one, masses of the gravitational excitons can correspond to energies which are achieved by present accelerators.

On the other hand, in the case of a non-constant internal scale factor background gravitational excitons interact with usual matter already in the lowest order approximation. If such interactions are strong enough then gravitational excitons cannot be considered as dark matter and they should contribute to the cross sections of usual particles. Equations (15) and (18) show for example that gravitational excitons can produce gravitons and vice versa. The form of interactions will depend on the concrete model. This should give a possibility to check experimentally different models on their compatibility with observational data. Possible interaction channels for tests could be e.g. interactions between gravitational excitons and abelian gauge fields or gravitational excitons and spinor fields [3].

Other interesting physical effects can be expected in the vicinity of topologically non-trivial objects such as black holes or cosmic strings, and in a multiply connected universe, e.g. in a universe with lorentzian wormholes (if they connect different regions of the same universe) or in a universe with a compact space manifold with negative or zero constant curvature. Propagating from a source to an observer on different sides of the topologically non-trivial object gravitational excitons can via  $SO(n)$ -rotation of the polarization vector in the target space acquire different polarizations (similar to  $SO(2) \approx U(1)$ -Aharonov-Bohm—phase-rotations in QED). In the observation region this will result in a local interference. Allowing the gravitational excitons to interact with other fields the interference picture should be observable.

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